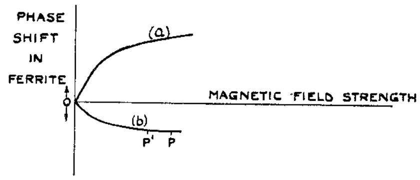

Fig. 3—Phase shift vs applied field. (a) Positively polarized wave. (b) Negatively polarized wave.

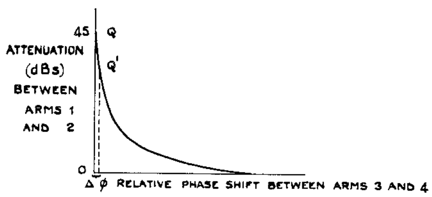

Fig. 4—Attenuation vs relative phase shift.

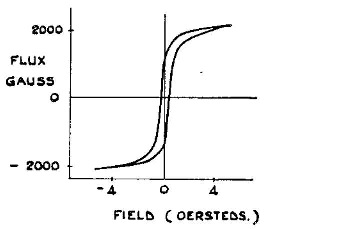

Fig. 5—Hysteresis loop of typical microwave ferrite.

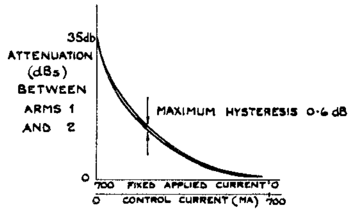

Fig. 6—Attenuator characteristic.

corresponds to the point Q in Fig. 4. If the magnetic field on the ferrite is now changed by a relatively large amount (to point P' in Fig. 3), the actual change in phase in arm 4 is quite small. This results in a small relative phase shift between arms 3 and 4 and this in turn causes the attenuation between arms 1 and 2 to change to the amount corresponding to Q' (Fig. 4). Thus it is obvious that on plotting a curve of attenuation between arms 1 and 2 vs magnetic field applied to the ferrite, a characteristic is obtained whose slope near the maximum attenuation point Q is considerably less steep than that of the curve of Fig. 4. This is shown in Fig. 6.

Since hysteresis is very small near saturation its effect near the steep part of the characteristic of Fig. 6 is very small. Below saturation the hysteresis of the ferrite is more marked (Fig. 5), but since the slope of the characteristic of Fig. 6 is much smaller when the applied field decreases, the effect of this increase in hysteresis is minimized. The final curve for the attenuator is shown in Fig. 6 where it can be seen that the maximum hysteresis measured corresponds to 0.6 db.

The author wishes to thank the manager, the English Electric Company, Ltd., Luton, for permission to publish this note.

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Quarter-Wave Compensation of Resonant Discontinuities*

Introduction

In designing RF transmission line components it is often necessary to place a short-circuited quarter-wavelength stub in parallel with the line or an open-circuited quarter-wavelength stub in series with the line. The stub can be broadbanded by merely changing the characteristic impedance of the line on either side of the stub for a distance of one quarter wavelength.

Broad-Band Stub

It is not generally recognized how broadband a simple stub with quarter-wave transformers can be made. Previous investigators[1] have merely adjusted the transformer impedance for perfect match at two frequencies which depart somewhat from the resonant frequency of the stub without regard for the reflection in the pass band. The following analysis tries to correlate the bandwidth with the allowable reflection in the pass band.

A coaxial broad-band stub is shown in Fig. 1. On each side of the stub the center conductor is enlarged for a length of $\lambda_0/4$ at the center frequency. In these quarter-wave transformers the characteristic impedance is Z_1. The stub is $\lambda_0/4$ long and its characteristic impedance is Z_2. The characteristic impedance of the line is taken as 1 ohm in what follows so that Z_1 and Z_2 are multiples of the characteristic impedance.

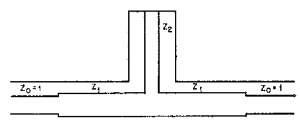

Fig. 1—Coaxial broad-band stub.

The $ABCD$ matrix of the stub plus transformers is

$$\begin{bmatrix} \cos\theta & jZ_1\sin\theta \\ j(1/Z_1)\sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -j(1/Z_2)\operatorname{ctn}\theta & 1 \end{bmatrix} \cdot \begin{bmatrix} \cos\theta & jZ_1\sin\theta \\ j(1/Z_1)\sin\theta & \cos\theta \end{bmatrix}$$

where θ is electrical length of each quarter-wave transformer and the stub. If we let $\theta=\pi/2+\phi$, the over-all matrix becomes

$$\begin{bmatrix} -\sin\phi & jZ_1\cos\phi \\ j(1/Z_1)\cos\phi & -\sin\phi \end{bmatrix} \begin{bmatrix} 1 & 0 \\ j(1/Z_2)\tan\phi & 1 \end{bmatrix} \cdot \begin{bmatrix} -\sin\phi & jZ_1\cos\phi \\ j(1/Z_1)\cos\phi & -\sin\phi \end{bmatrix}$$

which when multiplied gives

$$\begin{Bmatrix} \sin^2\phi-\cos^2\phi+(Z_1/Z_2)\sin^2\phi & -jZ_1\sin\phi\cos\phi(2+Z_1/Z_2) \\ -j(1/Z_1)\sin\phi\cos\phi(2-Z_1/Z_2\tan^2\phi) & \sin^2\phi-\cos^2\phi+(Z_1/Z_2)\sin^2\phi \end{Bmatrix}.$$

The insertion loss is given by[2]

$$\begin{aligned} L &= 10\log_{10}\{1+1/4[(A-D)^2-(B-C)^2]\} \\ &= 10\log_{10}\{1+1/4[(2/Z_1-2Z_1-Z_1^2/Z_2 \\ &\qquad +1/Z_2)\sin\phi\cos\phi-1/Z_2\tan\phi]^2\} \\ &= 10\log_{10}(1+m^2/4) \qquad (1) \end{aligned}$$

where

$$mZ_2 = R\sin\phi\cos\phi-\tan\phi \qquad (2)$$

$$R = 2Z_2/Z_1-2Z_1Z_2-Z_1^2+1. \qquad (3)$$

A graph of the magnitude of $|m|Z_2$ is shown in Fig. 2.

$R=1$ gives the maximally flat case with a zero derivative at the origin.

For R greater than 1, a triple peaked response is obtained.

Using some simple trigonometric substitutions it can be shown that $\phi_2=2\phi_1$; also,

$$m_1Z_2 = \tan\phi_2\left(\frac{1-\cos\phi_2}{1+\cos\phi_2}\right) \qquad (4)$$

and

$$R = \frac{2}{\cos^2\phi_2+\cos\phi_2} \qquad (5)$$

where ϕ_1 is the value of ϕ for worst reflections in the pass band, ϕ_2 is the band edge, and m_1 is the worst value of m in the pass band.

The quantity m_1 is related to the worst voltage standing wave ratio S by

$$m_1 = \frac{S-1}{\sqrt{S}} \qquad (6)$$

and the bandwidth is given by

$$BW = 2\phi_2/90. \qquad (7)$$

A graph of m_1Z_2 as a function of bandwidth is shown in Fig. 3.

As an example, suppose it is desired to design a stub support for a coaxial line to have a standing wave ratio of no greater than 1.05 over as wide a frequency band as possible. Because of voltage breakdown considerations it is decided that the largest value Z_2 may have is one. Then from (6), $m_1=0.0488$, and from Fig. 3 the bandwidth is 70.4 per cent or a frequency ratio of 2.09:1. R is determined from (5), and Z_1 from (3).

The required value of Z_1 for various values of Z_2 is plotted as a function of bandwidth in Fig. 4. This graph shows that the diameter of the quarter-wave transformers is rather critical. The desired Z_1 is only slightly smaller than the zero bandwidth case.

* Received by the PGMTT, October 27, 1958. The research in this document was supported jointly by the Army, Navy, and Air Force under contract with Mass. Inst. Tech.

[1] G. L. Ragan, "Microwave Transmission Circuits," M.I.T. Rad. Lab. Ser., McGraw-Hill Book Co., Inc., New York, N. Y., vol. 9, pp. 173–176; 1948.

[2] R. M. Fano and A. W. Lawson, "Microwave Transmission Circuits," M.I.T. Rad. Lab. Ser., McGraw-Hill Book Co., Inc., New York, N. Y., vol. 9, ch. 9 and 10; 1948.

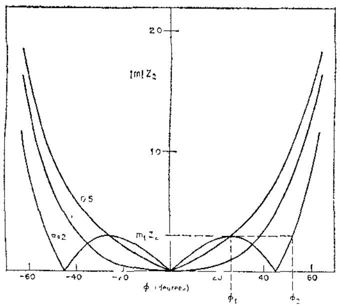

Fig. 2—Variation of m with ϕ for various values of R.

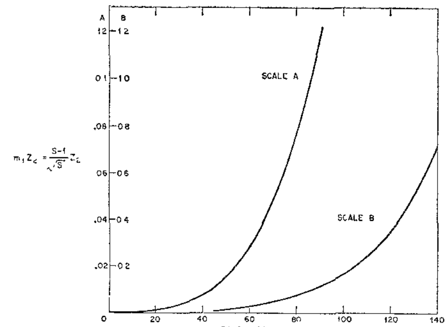

Fig. 3—Variation of m_1Z_2 with bandwidth.

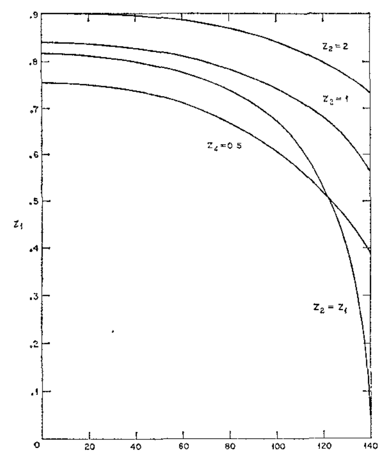

Fig. 4—Z_1 as a function of bandwidth for various stub impedances.

Although the above analysis assumed a coaxial stub it is obvious that the analysis can be applied to other TEM transmission lines and to waveguides. In particular it may be applied to the problem of making an extremely broad-band T junction for a branched duplexer.

An Extremely Broad-Band Rotary Joint

Electrically a choke type rotary joint consists of an open-circuited quarter-wavelength stub in series with a transmission line. A comparison with the broad-band stub of the previous section, which is a short-circuited quarter-wavelength stub in parallel with the line, suggests that an analysis of the choke type rotary joint on an admittance basis may give a result similar to the above analysis. This indeed is the case. If Z_1 and Z_2 in (1) through (7) are replaced by Y_1 and Y_2, respectively, these equations will give the response of a choke type rotary joint and at the same time will point out a method of broadbanding such a rotary joint. Broadbanding may be achieved by reducing the characteristic admittance of the transmission line by the proper amount for a quarter wavelength on either side of the chokes. Physically, this may be accomplished by decreasing the radius of the inner conductor or increasing the radius of the outer conductor of a coaxial line rotary joint.

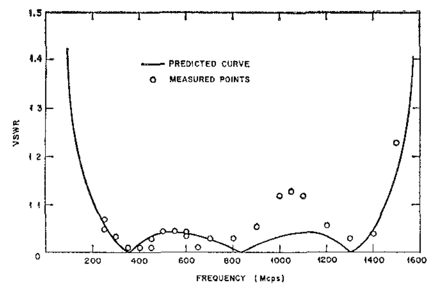

Fig. 5—Predicted and measured VSWR of broad-band rotary joint.

A broad-band rotary joint using the above theory has been built and tested in three and one-eighth inch coaxial line. The predicted and measured results are shown in Fig. 5. The measured results agree quite well with the theory except in the region near 1100 mc. This can be explained by the lack of the theory in accounting for the capacitive discontinuity at the end of the series choke in the inner conductor and the effect of the short-circuited high impedance quarter-wave section at the end of the series choke in the outer conductor.

The sum of the characteristic impedances of the inner and outer chokes was 3.3 ohms and the main line had an impedance of 50 ohms. The rotary joint was designed to have a VSWR less than 1.04 over a 135 per cent bandwidth. For the same VSWR with no compensation the bandwidth would have been 70 per cent.

In conclusion the above analysis may be used to broadband any quarter-wavelength choke or stub type discontinuity and accurately predict its performance.

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Comments on Ozaki's Comments*

Ozaki's[1] comments have drawn my attention to the fact that there is a significant difference between "The Synthesis Theorem" given in my paper[2] and the restatement of it that is given and proven in the Appendix. If we define l to be the degree, the maximum of the degrees of the numerator and the denominator, of the rational function $Z(p)$, and r to be the number of line sections in the impedance transformer, then in the first statement of the theorem, l is unspecified and $n=r$ while in the second statement $l=n$ and r is unspecified. Now the second theorem is correct, even in view of Ozaki's comments, and accordingly is adequate for a proof of the physical realizability of the allowed insertion loss functions. The first theorem, however, is incorrectly stated as Ozaki's example has shown.

Ozaki's third condition, "Assuming that the numerator and denominator of $Z(p)$ in (1) are prime to each other, the degrees of both the numerator and denominator must be equal to n," correctly requires that $l=n$ and adds the restriction that the numerator and denominator of $Z(p)$ contain no common factors.

The requirement that the degree of the numerator of $Z(p)$ equal the degree of the denominator is a salient feature of the theory. My failure to define $l=n$, which has this consequence when taken with condition 2, in the first statement of the theorem, was simply an oversight. I permitted the removal of common factors[3] in the second statement of the theorem by not specifying r, since it is readily shown that the removal of a common factor from the numerator and denominator of a $Z(p)$, satisfying condition 2, results in a $Z'(p)$ which again satisfies this condition

Ozaki's third condition permits the proof of a sharper theorem than my application required, namely one in which $l=n=r$. His condition, however, is unnecessarily restrictive since the relative primeness of the numerator and denominator is not a necessary condition for the truth of this class of theorem. For example, if the terminating resistance, R, is preceded by a section of line of characteristic impedance, R, then the numerator and denominator of $Z(p)$ contain the common factor, $p+1$. In fact it is readily demonstrated that the only common factors permitted by condition 2 are products of $p+1$ and $p-1$. The first can be realized while the occurrence of the latter would result in the indeterminacy of $Z(1)$.

A more general theorem of this type can be stated:

The necessary and sufficient conditions that a rational function of p, determinant for $p=1$, with real coefficients, of degree at most n in numerator or denominator written in the form

$$Z(p) = \frac{m_1(p) + n_1(p)}{m_2(p) + n_2(p)}$$

with m_1 and m_2 odd or even and n_1 and n_2 even or odd, be the input impedance of a cascade of n equal-length transmission line sections terminated in a resistance are:

* Received by the PGMTT, October 30, 1958.

[1] H. Ozaki, "On Riblet's theorem," IRE Trans. on Microwave Theory and Techniques, vol. MTT-6, pp. 331–332; July, 1958.

[2] H. J. Riblet, "General synthesis of quarter-wave impedance transformers," IRE Trans. on Microwave Theory and Techniques, vol. MTT-5, pp. 36–43; January, 1957.

[3] For example, the well-known result that a positive real function of p is a quotient of two Hurwitz polynomials is true in general only if the removal of common factors from numerator and denominator is permitted.